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QCD condensates with flavor SU(3) symmetry breaking from the instanton vacuum

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Abstract

We investigate the effects of flavor SU(3)-symmetry breaking on the quark, gluon, and mixed quark-gluon condensates, based on the nonlocal effective chiral action from the instanton vacuum. We take into account the effects of the flavor SU(3) symmetry breaking in the effective chiral action, so that the dynamical quark mass depends on the current quark mass (m_f). We compare the results of the present approach with those without the current quark mass dependence of the dynamical quark mass. It is found that the result of the quark condensate is decreased by about 30 % as m_f increases to 200 MeV, while that of the quark-gluon mixed condensates is diminished by about 15%. We obtain the ratios of the quark and quark-gluon mixed condensates, respectively: $[\langle\bar{s}s\rangle/\langle\bar{u}u\rangle]^{1/3} = 0.75$ and $[\langle\bar{s}\sigma_{\mu\nu}G^{\mu\nu}s\rangle/\langle\bar{u}\sigma_{\mu\nu}G^{\mu\nu}u\rangle]^{1/5} = 0.87$. It turns out that the dimensional parameter $m_0^2 = \langle\bar{q}\sigma_{\mu\nu}G^{\mu\nu}q\rangle/\langle\bar{q}q\rangle = 1.60 \sim 1.92\,\mathrm{GeV}^2$.

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1. Understanding the QCD vacuum is very complicated, since both perturbative and non-perturbative fluctuations come into play. In particular, the quark and gluon condensates, being the lowest dimensional ones, characterize the non-perturbative structure of the QCD vacuum. The quark condensate is identified as the order parameter for spontaneous chiral-symmetry breaking (SχSB) which plays an essential role in describing low-energy phenomena of hadrons: In the QCD sum rule, these condensates come from the operator product expansion and are related to hadronic observables [1], while in chiral perturbation theory (χPT), the free parameter B_0 is introduced in the mass term of the effective chiral Lagrangian at the leading order [2] measuring the strength of the quark condensate [3]. On the other hand, the gluon condensate is not the order parameter but measures the vacuum energy density [1], which was first estimated by the charmonium sum rule [4].

While the quark and gluon condensates are well understood phenomenologically, higher dimensional condensates suffer from large uncertainty. Though it is still possible to estimate dimension-six four-quark condensates in terms of the quark condensate by using the factorization scheme which is justified in the large N_c limit, the dimension-five mixed quark-gluon condensate is not easily determined phenomenologically. In particular, the mixed condensate is an essential parameter to calculate baryon masses [5], exotic hybrid mesons [6], higher-twist meson distribution amplitudes [7] within the QCD sum rules. Moreover, the mixed quark condensate plays a role of an additional order parameter for SχSB, since the quark chirality flips via the quark-gluon operator. Thus, it is naturally expressed in terms of the quark condensate:

$$\langle\bar{\psi}\sigma^{\mu\nu}G_{\mu\nu}\psi\rangle = m_0^2\langle\bar{\psi}\psi\rangle \tag{1}$$

with the dimensional parameter m_0^2 which was estimated in various works [5, 8, 9, 10, 11, 12] and with the definition $G_{\mu\nu} = G^a_{\mu\nu}\lambda^a/2$.

The instanton picture allows us to study the QCD vacuum microscopically. Since the instanton picture provides a natural mechanism for $S\chi$SB due to the delocalization of single-instanton quark zero modes in the instanton medium, the quark condensate can be evaluated. The instanton vacuum is validated by the two parameters: The average instanton size $\bar{\rho} \sim 1/3\,\mathrm{fm}$ and average inter-instanton distance $R \sim 1\,\mathrm{fm}$. These essential numbers were suggested by Shuryak [13] within the instanton liquid model and were derived from $\Lambda_{\overline{\mathrm{MS}}}$ by Diakonov and Petrov [14]. These values were recently confirmed by lattice QCD simulations [15, 16, 17, 18, 19].

In the present work, we want to investigate the effect of flavor SU(3)-symmetry breaking on the above-mentioned three QCD condensates, *i.e.* quark, gluon, and mixed quark-gluon condensates, based on the instanton liquid model for the QCD vacuum [20, 21, 22]. The model was later extended by introducing the current quark masses [23, 24, 25, 26, 27, 28]. Since we are interested in the effect of explicit flavor SU(3)-symmetry breaking, we follow the formalism of Ref. [23] in which the dependence of the dynamical quark mass on the current quark mass has been studied in detail. Though the mixed quark-gluon condensate was already studied in the instanton vacuum [11], explicit SU(3)-symmetry breaking was not considered. Hence, we want now to extend the work of Ref. [11], emphasizing the effect of flavor SU(3)-symmetry breaking on the QCD vacuum condensates. We will show that with a proper choice of the m_f dependence of the dynamical quark mass [29] the gluon condensate is independent of m_f. The corresponding results are summarized as follows: The ratio $[\langle\bar{s}s\rangle/\langle\bar{u}u\rangle]^{1/3} = 0.75$, $[\langle\bar{s}\sigma_{\mu\nu}G^{\mu\nu}s\rangle/\langle\bar{u}\sigma_{\mu\nu}G^{\mu\nu}u\rangle]^{1/5} = 0.87$, $m^2_{0,\mathrm{u}} = 1.60\,\mathrm{GeV}^2$, and $m^2_{0,\mathrm{s}} = 1.84\,\mathrm{GeV}^2$, with isospin symmetry assumed.

2. We start with the effective low-energy QCD partition function from the instanton vacuum with SU(3)-symmetry breaking taken into account [23, 24, 25]:

$$\mathcal{Z} \;=\; \int D\psi^\dagger D\psi \exp\left[\int d^4x \sum_f \psi_f^\dagger(i\partial\!\!\!/ + im_f)\psi_f\right]\left[\frac{Y_+^{N_f}}{VM^{N_f}}\right]^{N_+}\left[\frac{Y_-^{N_f}}{VM^{N_f}}\right]^{N_-}, \tag{2}$$

where ψ_f and $\psi_f^\dagger$ denote the quark fields and m_f stands for the current quark mass for a given flavor. We consider here the strict large N_c expansion [1]. V and M^{N_f} stand for the space-time volume and for the dimensional parameter, respectively. Note that, by dropping the current quark mass m_f in the first bracket in the r.h.s., Eq. (2) turns out to be the same as that derived in the chiral limit [20, 21]. $Y_\pm$ in Eq. (2) represents a 't Hooft-type $2N_f$-quark interaction generated by instantons:

$$\begin{aligned} Y_\pm^{N_f} \;=&\; \int d\rho d(\rho)\int dU\int d^4x\prod_f\int\frac{d^4k_f}{(2\pi)^4}\frac{d^4p_f}{(2\pi)^4}[2\pi\rho F_f(k_f)][2\pi\rho F_f(p_f)] \\ &\times\;\exp\left[-x\cdot\left(\sum_f k_f-\sum_f p_f\right)\right]\left[U^\alpha_{i'}(U^{j'}_\beta)^\dagger\epsilon^{ii'}\epsilon_{jj'}\right]_f\left[i\psi_f^\dagger(k_f)_{\alpha i}\frac{1\pm\gamma_5}{2}\psi_f(p_f)^{\beta j}\right], \end{aligned} \tag{3}$$

where ρ denotes the instanton size and U represents the color orientation matrix. Since we are interested in the vacuum condensates, it is enough to consider the case of $N_f = 1$ in which the integration over U becomes trivial. We employ the instanton distribution of a delta-function type, so that we get a simple quark-quark interaction for a given flavor f:

$$Y_\pm = \frac{i}{N_c}\int\frac{d^4k}{(2\pi)^4}[2\pi\bar\rho F_f(k\bar\rho)]^2\left[\psi_f^\dagger(k)\frac{1\pm\gamma_5}{2}\psi_f(k)\right]. \tag{4}$$

$F_f(k)$ is the Fourier transformation of the fermionic zero mode solution $\Phi_{I\bar I}$. We implicitly assumed that $F_f(k)$ is a function of the current-quark mass. This assumption will be verified in what follows. $\Phi_{I\bar I}$ satisfies the following Dirac equation under the (anti)instanton effects $A_{I\bar I}$:

$$[i\partial\!\!\!/ + A\!\!\!/_{I\bar I} + im_f]\,\Phi_{I\bar I} = 0. \tag{5}$$

Instead of computing $\Phi_{I\bar I}$ directly from Eq. (5), being equivalently, we follows the course suggested in Ref. [29] in which Pobylitsa used a elaborated and systematic expansion of the quark propagator $\langle x|(i\partial\!\!\!/ + A\!\!\!/_I + im_f)^{-1}|x\rangle$. By doing this, one can immediately obtain analytical form of $F_f(k)$. Here, we make a brief explanation on this method. Diakonov *et al.* made a zero-mode approximation for the quark propagator in the instantons [21]:

$$\left\langle x\left|\frac{1}{i\partial\!\!\!/ + A\!\!\!/_I + im}\right|y\right\rangle \simeq \left\langle x\left|\frac{1}{i\partial\!\!\!/ + im}\right|y\right\rangle + \frac{\Phi_I(x)\Phi_I^\dagger(y)}{im}. \tag{6}$$

We note that this zero-mode approximation causes one difficult problem of breaking the conservation of vector and axial-vector currents. To amend this current conservation problem,

[1] Recently, it is shown that the effects of mesonic loops on the quark condensate, which is the next-to-leading order in the N_c expansion, is not small [30]. However, a proper adjustment of the parameters $\bar\rho$ and R may yield approximately the same results as those without the meson-loop corrections.

Pobylitsa expand the quark propagator as follows. Having summed up the planar diagrams, one arrives the following integral equation for a quark propagator:

$$
\begin{aligned}
S &= \left\langle x \left| \frac{1}{i\partial\!\!\!/ + A\!\!\!/_I + im} \right| x \right\rangle \\
&= \left\langle x \left| \frac{1}{i\partial\!\!\!/ + im} \right| x \right\rangle + \frac{N}{2VN_c}\mathrm{tr}_c \left[\int d^4 Z_I \left[\langle x \left| (i\partial\!\!\!/ + A\!\!\!/_I + im) \right| x \rangle - \frac{1}{A\!\!\!/} \right]^{-1} + (I \to \bar{I}) \right], \quad (7)
\end{aligned}
$$

where $Z_{I\bar{I}}$ indicates the collective coordinate for the (anti)instanton. Then the inverse of the quark propagator can be approximately written with an arbitrary scalar function to be determined:

$$
S^{-1}(i\partial) = i\partial\!\!\!/ + im_f + iM_0 F_f^2(i\partial), \quad M_0 = \frac{\lambda\,[2\pi\bar{\rho}]^2}{N_c}. \qquad (8)
$$

Note that we choose the scalar function to be $M_0 F_f^2(i\partial)$ for later convenience. The value of M_0 will be determined by the self-consistent equation (so called "saddle-point equation") of the model. To obtain $F_f(k)$, Inserting Eq.(8) into Eq.(7), we obtain an integral equation for $F_f(i\partial)$:

$$
iF_f^2(i\partial) \simeq M_0 \mathrm{tr}_c \left[\int d^4 Z_I A\!\!\!/_I \left(A\!\!\!/_I + i\partial\!\!\!/ + im_f - iM_0 F_f^2(i\partial) \right)^{-1} (i\partial\!\!\!/ + im_f) + (I \to \bar{I}) \right]. \qquad (9)
$$

Having carried out a straightforward manipulation, finally, we arrive at:

$$
\begin{aligned}
M_f(k) &= M_0 F_f^2(k) \left[\sqrt{1 + \frac{m_f^2}{d^2}} - \frac{m_f}{d} \right] = M_0 F_f^2(k) f(m_f), \quad d = \sqrt{\frac{0.08385}{2N_c}} \frac{8\pi\bar{\rho}}{R^2} \\
F_f(k) &= 2t \left[I_0(t)K_1(t) - I_1(t)K_0(t) - \frac{1}{t} I_1(t)K_1(t) \right] \simeq 0.198\,\mathrm{GeV}, \quad t = |k|\bar{\rho}/2. \qquad (10)
\end{aligned}
$$

Thus, we have exact form of $F_f(k)$ in terms of explicitly broken flavor SU(3) symmetry ($m_f \neq 0$).

Now, we are in position to discuss the saddle-point equation of the model derived from the effective QCD partition function. Introducing the Lagrange multipliers $\lambda_\pm$ [22], we are able to exponentiate the partition function of Eq. (2):

$$
\begin{aligned}
\mathcal{Z} &= \int \frac{d\lambda_\pm}{2\pi} \int D\psi^\dagger D\psi \exp \left[\int d^4 x \bar{\psi}_f (i\partial\!\!\!/ + im_f)\psi_f \right] \\
&\times \exp \left[N_+ \left(\ln \frac{N_+}{\lambda_+ V M^{N_f}} - 1 \right) + \lambda_+ Y_+ \right] \exp \left[N_- \left(\ln \frac{N_-}{\lambda_- V M^{N_f}} - 1 \right) + \lambda_- Y_- \right]. \qquad (11)
\end{aligned}
$$

In the following, we assume that $N_+ = N_- = N/2$ and $\lambda_+ = \lambda_- = \lambda$ so that the partition function can be simplified as follows:

$$
\mathcal{Z} = \int \frac{d\lambda}{2\pi} \int D\psi^\dagger D\psi \exp \left[\int d^4 x \psi_f^\dagger (i\partial\!\!\!/ + im_f)\psi_f + N \left(\ln \frac{N}{2\lambda V M^{N_f}} - 1 \right) + \lambda (Y_+ + Y_-) \right]. (12)
$$

It is also interesting to compare Eq. (12) with the partition function proposed by Diakonov *et al.*:

$$
\mathcal{Z}_D = \int \frac{d\lambda}{2\pi} \int D\psi^\dagger D\psi \exp \left[\int d^4 x \psi_f^\dagger i\partial\!\!\!/ \psi_f + N \left(\ln \frac{N}{2\lambda V M^{N_f}} - 1 \right) + \lambda (Y_+ + Y_- + 2m_f) \right]. (13)
$$

One can easily find the difference between the effective actions of Eq. (12) and of Eq. (13); In Eq. (12), the current quark mass m_f appears as a mass term of the quark fields accounting for the explicit breaking of chiral symmetry, while the m_f is placed in the quark-instanton vertex in Eq. (13). We note that this configuration of the current quark mass is deeply related to the parity-violating quark mass term ($\propto \psi^\dagger\gamma_5\psi$) in terms of the instanton number fluctuation [21].

Concentrating on the first and third brackets in Eq. (12), we get the fermionic trace log which relates to the quark propagator as follows:

$$\int d^4x \int \frac{d^4k}{(2\pi)^4}\mathrm{tr}_{c\gamma} \ln \frac{\left[-\not{k}+im_f+\frac{i\lambda}{N_c}[2\pi\bar{\rho}F_f(k\bar{\rho})]^2\right]}{[-\not{k}+im_f]}$$
$$= N_c V \int \frac{d^4k}{(2\pi)^4}\mathrm{tr}_\gamma \ln \frac{[-\not{k}+im_f+iM_f(k)]}{[-\not{k}+im_f]}, \tag{14}$$

where the subscripts c and γ denote the color and Dirac-spin spaces. Thus, we obtain the partition function in a compact form as follows:

$$\mathcal{Z} = \int \frac{d\lambda}{2\pi} \exp\left[N\left(\ln\frac{N}{2\lambda V M^{N_f}}-1\right) + N_c V\int\frac{d^4k}{(2\pi)^4}\mathrm{tr}_d \ln\frac{[-\not{k}+im_f+iM_f(k)]}{[-\not{k}+im_f]}\right]. \tag{15}$$

Now, we perform a functional variation of the partition function with respect to λ:

$$\begin{aligned}
\frac{\delta \ln\mathcal{Z}}{\delta\lambda} &= -\frac{N}{\lambda} + N_c V\int\frac{d^4k}{(2\pi)^4}\mathrm{tr}_\gamma\frac{iM_f(k)/N_c}{-\not{k}+im_f+iM_f(k)} = 0,\\
\frac{N}{\lambda} &= N_c V\int\frac{d^4k}{(2\pi)^4}\mathrm{tr}_\gamma\frac{iM_f(k)[-\not{k}-im_f-iM_f(k)]}{k^2+[m_f+M_f(k)]^2},\\
\frac{N}{V} &= 4N_c\int\frac{d^4k}{(2\pi)^4}\frac{M_f(k)[m_f+M_f(k)]}{k^2+[m_f+M_f(k)]^2}.
\end{aligned} \tag{16}$$

Final formula in Eq. (16) is called the saddle-point equation of the model. We employ the standard values of the instanton ensemble, $N/V = 200^4$ MeV^4 and $\bar{\rho}\sim 1/3$ fm $\simeq 1/600$ MeV^{-1} for the numerical calculations. From these values, we obtain $M_0 = 0.350$ GeV, which is determined self-consistently by the saddle-point equation. Using this value, we obtain $i\langle u^\dagger u\rangle \simeq 250^3$ MeV^3 and pion weak decay constant $F_\pi \simeq 93$ MeV. We note that this saddle-point equation is closely related to the gluon-condensate [31]; $\langle G_{\mu\nu}G^{\mu\nu}\rangle_f = 32\pi^2 N/V$.

As for an arbitrary N_f, it becomes rather difficult to obtain the saddle-point equation, since we need to perform a complicated integration over the color orientation. However, keeping only the leading order in the large N_c limit and additional variation over the Hermitian $N_f\times N_f$ flavor matrix ($\mathcal{M}$ in Ref. [21]), we have the same form of the saddle-point equation for each flavor as shown in Eq. (16). The similar argument is also possible for the quark and mixed condensates [11, 21, 24].

Being similar to the saddle-point equation (the gluon condensate), the quark condensate for each flavor can be obtained by the following functional derivative with respect to the current quark mass m_f:

$$\begin{aligned}
\langle\bar{q}q\rangle_f &= \frac{1}{V}\frac{\delta\ln\mathcal{Z}}{\delta m_f} = -iN_c\int\frac{d^4k}{(2\pi)^4}\mathrm{tr}_\gamma\left[\frac{\not{k}+i[m_f+M_f(k)]}{k^2+[m_f+M_f(k)]^2} - \frac{\not{k}+im_f}{k^2+m_f^2}\right]\\
&= 4N_c\int\frac{d^4k}{(2\pi)^4}\left[\frac{m_f+M_f(k)}{k^2+[m_f+M_f(k)]^2}-\frac{m_f}{k^2+m_f^2}\right].
\end{aligned} \tag{17}$$

One can immediately see that when $m_f \to 0$, Eq. (17) gets equal to the well-known expression for the quark condensate in the chiral limit.

Now, we discuss how to calculate the mixed condensate, $\langle \bar{q}\sigma_{\mu\nu}G^{\mu\nu}q\rangle$. Actually, the local operator inside this condensate corresponds to the quark-gluon interaction of a Yukawa type. However, in the present work, the gluon field strength ($G_{\mu\nu}$) can be expressed in terms of the quark-instanton interaction [11, 21]. First, the one flavor quark and one instanton interaction in Eq. (3) can be rewritten as a function of space-time coordinates x and color orientation matrix U:

$$\begin{aligned} Y_{\pm,1}(x,U) &= \int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}[2\pi\rho F_f(k_1)][2\pi\rho F_f(k_2)]e^{-ix\cdot(k_1-k_2)} \\ &\times \left[U^{\alpha}_{i'}(U^{j'}_{\beta})^{\dagger}\epsilon^{ii'}\epsilon_{jj'}\right]_f \left[i\psi^{\dagger}_f(k_1)_{\alpha i}\frac{1\pm\gamma_5}{2}\psi_f(k_2)^{\beta j}\right]. \end{aligned} \tag{18}$$

Here, we assume again the delta function-type instanton distribution. We define then the field strength $G^a_{\mu\nu}$ in terms of the instanton configuration as a function of $I(\bar{I})$ position and orientation matrix U:

$$G^a_{\pm\mu\nu}(x,x',U) = \frac{1}{2}\left[\lambda^a U\lambda^b U^{\dagger}\right]G^b_{\pm\mu\nu}(x'-x). \tag{19}$$

$G^b_{\pm\mu\nu}(x'-x)$ stands for the field strength consisting of a certain instanton configuration. Using Eqs. (18) and (19), we define the field strength in terms of the quark-instanton interaction:

$$\hat{G}^a_{\pm\mu\nu} = \frac{iN_cM}{4\pi\bar{\rho}^2}\int d^4x\int dU G^a_{\pm\mu\nu}(x,x',U)Y_{\pm,1}(x,U) \tag{20}$$

Following the method in Ref. [11, 21], we finally obtain the quark-gluon mixed condensate as follows:

$$\langle \bar{q}\sigma_{\mu\nu}G^{\mu\nu}q\rangle_f = 2N_c\bar{\rho}^2\int\frac{d^4k_1}{(2\pi)^4}\int\frac{d^4k_2}{(2\pi)^4}\frac{\sqrt{M_f(k_1)M_f(k_2)}G(k_1,k_2)N(k_1,k_2)}{[k_1^2+[m_f+M_f(k_1)]^2][k_2^2+[m_f+M_f(k_2)]^2]}, \tag{21}$$

where $G(k_1,k_2)$ and $N(k_1,k_2)$ are defined as follows:

$$\begin{aligned} G(k_1,k_2) &= 32\pi^2\left[\frac{K_0(t)}{2}+\left[\frac{4K_0(t)}{t^2}+\left(\frac{2}{t}+\frac{8}{t^3}\right)K_1(t)-\frac{8}{t^4}\right]\right], \\ N(k_1,k_2) &= \frac{1}{(k_1-k_2)^2}\left[8k_1^2k_2^2-6(k_1^2+k_2^2)k_1\cdot k_2+4(k_1\cdot k_2)^2\right] \end{aligned} \tag{22}$$

with $t = |k_1-k_2|\bar{\rho}$. The functions K_0 and K_1 stand for the modified Bessel functions of the second kind of order 0 and 1, respectively. If we consider for arbitrary N_f the mixed condensate, the situation may be somewhat different from the cases of the gluon and quark condensates. We note that, though the mixed condensate has been calculated for the case of $N_f = 1$, the same formula of Eq. (21) still holds for each flavor with arbitrary N_f as discussed previously [11].

3. We first discuss the numerical results for the gluon condensate, $\langle G_{\mu\nu}G^{\mu\nu}\rangle/32\pi^2 = N/V$. As mentioned before, we use $N/V = 200^4$ MeV^4 [31]. In Fig. 1, we show the

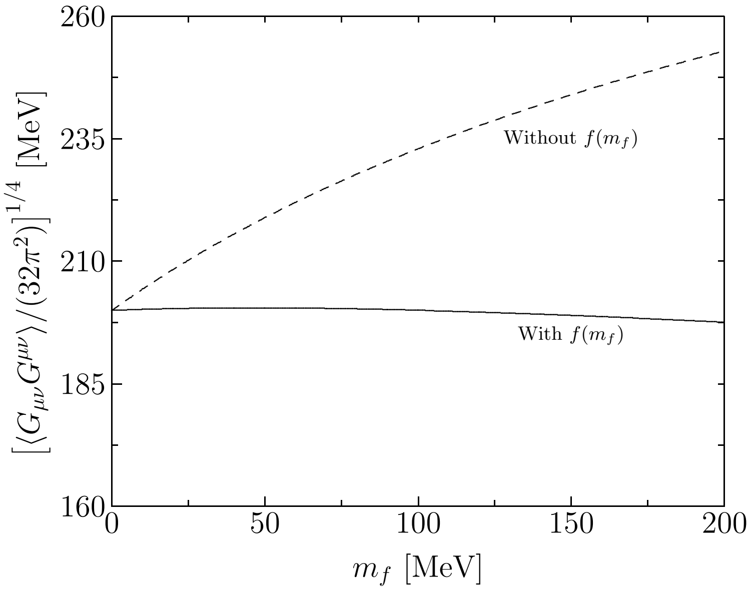

FIG. 1: Gluon condensate as a function of the current quark mass m_f. The solid curve draws the gluon condensate with the m_f-correction factor $f(m_f)$ in Eq. (10) and the dashed one corresponds to that without it.

numerical results of the gluon condensates with and without the m_f-correction factor $f(m_f)$ in Eq. (10). As shown in Fig. 1, the result with $f(m_f)$ is noticeably different from that without it. Since the gluon condensate is related to the vacuum energy density, it should be independent of the current quark mass m_f. However, if we turn off the m_f-correction factor, the gluon condensate increases almost linearly. It indicates that it is essential to have the m_f-correction factor $f(m_f)$ in order to consider SU(3) flavor symmetry breaking properly. In Table I we list the values of the gluon condensate when $m_f = m_u = 5$ MeV and $m_f = m_s = 150$ MeV with and without $f(m_f)$.

In Fig. 2, we depict the results of the quark condensate in a similar manner. In general, the quark condensate decreases as the value of m_f increases, as already shown in Ref. [30]. The values of the quark condensate are listed in Table II. As for the u-quark condensate, we have $-250^3 \sim -260^3$ MeV^3. Here, we assume isospin symmetry for the light quarks; $m_u = m_d = 5$ MeV. We find that the nonstrange quark condensate does not depend much on $f(m_f)$. In contrast, the strange one is sensitive to the m_f correction factor as shown in Table II (and also in Fig. 2). We find that without the $f(m_f)$ the quark condensate is decreased by about 10 % as m_f increases from 0 to 200 MeV, while it is diminished by about 30 % with the $f(m_f)$. Being compared with the results of Refs. [9, 23, 32], the quark condensate in the present work is obtained to be similar to them with the correction factor.

	With $f(m_f)$	without $f(m_f)$
$\langle G_{\mu\nu}G^{\mu\nu}\rangle_u/32\pi^2$	200^4	202^4
$\langle G_{\mu\nu}G^{\mu\nu}\rangle_s/32\pi^2$	199^4	243^4

TABLE I: Gluon condensates for $m_u = 5$ and $m_s = 150$ MeV [MeV^4].

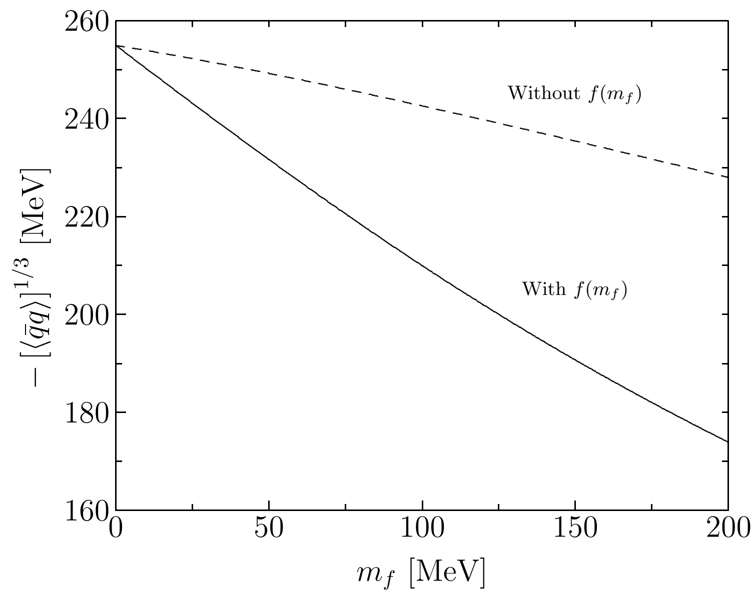

FIG. 2: Quark condensate as a function of the current quark mass m_f. The solid curve draws the quark condensate with the m_f-correction factor $f(m_f)$ in Eq. (10) and the dashed one corresponds to that without it.

In Fig. 3, we draw the results of the quark-gluon mixed condensate as functions of the m_f. The correction factor $f(m_f)$ being taken into account, the mixed condensate is decreased by about 15 %. However, it is almost constant when $f(m_f)$ is switched off. We also list the values of the mixed condensate for the up and strange quarks in Table III.

We now consider the effect of flavor SU(3)-symmetry breaking by calculating the ratios between the nonstrange condensates and the strange ones. As already discussed, the ratio of the gluon condensates remains unity for the m_f-correction factor. As for the ratio of the quark condensate, $[\langle\bar{s}s\rangle/\langle\bar{u}u\rangle]^{1/3}$, there is a great amount of theoretical calculations and those results lie in the range: $0.79 \sim 1.03$ [9, 23, 32]. Our results are $0.75 \sim 0.93$ for the quark condensate, which is consistent with them. We note that $[\langle\bar{s}s\rangle/\langle\bar{u}u\rangle]^{1/3} = 0.75$ with the m_f-correction factor will be taken as our best result. Being compared to Refs. [34, 35, 36, 37], our results are in good agreement with them.

We now study a dimensional quantity m_0^2 defined as the ratio between the mixed and quark condensates:

$$m_0^2 = \langle\bar{q}\sigma_{\mu\nu}G^{\mu\nu}q\rangle/\langle\bar{q}q\rangle, \tag{23}$$

which is an important input for general QCD sum rule calculations. We draw m_0^2 in Fig. 4

	With $f(m_f)$	without $f(m_f)$
$\langle\bar{u}u\rangle$	-253^3	-254^3
$\langle\bar{s}s\rangle$	-191^3	-235^3

TABLE II: Quark condensates for $m_u = 5$ and $m_s = 150$ MeV [MeV^3].

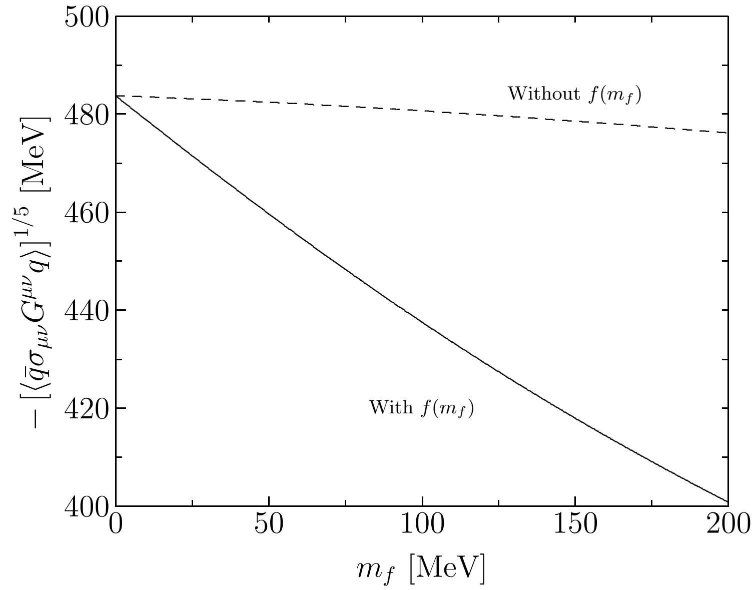

FIG. 3: Quark-gluon mixed condensate as a function of the current quark mass m_f. The solid curve draws the mixed condensate with the m_f-correction factor $f(m_f)$ in Eq. (10) and the dashed one corresponds to that without it.

as a function of m_f and list the values of m_0^2 in Table IV. The value of m_0^2 increases as m_f does, which implies that the mixed condensate is less sensitive to the m_f than the quark condensate. The values of m_0^2 are in the range of $1.84\,\mathrm{GeV}^2$ for the strange quark and of $1.60\,\mathrm{GeV}^2$ for the up quark. Thus, the strange $m_{0,s}^2$ turns out to be larger than the nonstrange $m_{0,u}^2$ by about 15 %.

We now consider the scale dependence of the condensates.

$$\langle\mathcal{O}_1(\mu_A)\rangle = \left[\frac{\alpha_s(\mu_B)}{\alpha_s(\mu_A)}\right]^{\gamma_1/b}\langle\mathcal{O}_1(\mu_B)\rangle,\ \ \langle\mathcal{O}_2(\mu_A)\rangle = \left[\frac{\alpha_s(\mu_B)}{\alpha_s(\mu_A)}\right]^{\gamma_2/b}\langle\mathcal{O}_2(\mu_B)\rangle, \tag{24}$$

where the subscripts 1 and 2 indicate different generic operators corresponding to the condensates in which we are interested. The μ_A and μ_B denote two different renormalization scales of the operators. γ is the corresponding anomalous dimension for the condensates. b is defined as $11N_c/3 - 2N_f/3$ and becomes $11 - 2 = 9$ in the present work. Taking the ratio of the operators 1 and 2, we have:

$$\frac{\langle\mathcal{O}_1(\mu_A)\rangle}{\langle\mathcal{O}_2(\mu_A)\rangle} = \left[\frac{\alpha_s(\mu_B)}{\alpha_s(\mu_A)}\right]^{(\gamma_1-\gamma_2)/b}\frac{\langle\mathcal{O}_1(\mu_B)\rangle}{\langle\mathcal{O}_2(\mu_B)\rangle}. \tag{25}$$

	With $f(m_f)$	without $f(m_f)$
$\langle\bar{u}\sigma_{\mu\nu}G^{\mu\nu}u\rangle$	-481^5	-484^5
$\langle\bar{s}\sigma_{\mu\nu}G^{\mu\nu}s\rangle$	-418^5	-483^5

TABLE III: Quark-gluon mixed condensates for $m_u = 5$ and $m_s = 150$ MeV [MeV^5].

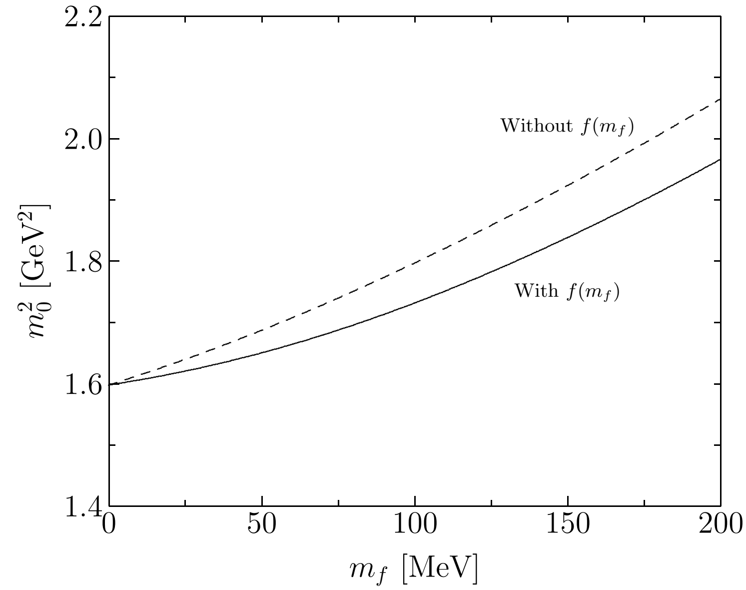

FIG. 4: Ratio m_0^2 as a function of the current quark mass m_f. The solid curve draws the gluon condensate with the m_f-correction factor $f(m_f)$ in Eq. (10) and the dashed one corresponds to that without it.

If $\mathcal{O}_1$ and $\mathcal{O}_2$ stand for the mixed and quark condensates respectively, we have $\gamma_1 = -2/3$ and $\gamma_2 = 4$ resulting in $(\gamma_1 - \gamma_2)/b = -14/27 \simeq -0.52$. On the contrary, we have $\gamma_1/b = -2/27 \simeq -0.07$ and $\gamma_2/b = 4/9 \simeq 0.44$. Thus, m_0^2 shows relatively strong dependence on the scale, whereas the quark and mixed condensates are less influenced by the scaling. Thus, m_0^2 depends more strongly on the scale.

Finally, we would like to mention that the relation between the mixed condensate and the expectation value of the transverse momentum for the leading-twist light-cone distribution amplitude. This relation can be written as follows:

$$\langle k_T^2 \rangle_\pi = \frac{5}{36} \frac{\langle \bar{u}\sigma_{\mu\nu} G^{\mu\nu} u\rangle}{\langle \bar{u}u\rangle} = \frac{5m_0^2}{36}. \tag{26}$$

Considering m_0^2 for the light quarks in Table IV ($m_0^2 = 0.16\,\mathrm{GeV}^2$), we get $\langle k_T^2\rangle_\pi = 0.2\,\mathrm{GeV}^2$. Note that the value estimated here is in qualitatively good agreement with that calculated

	With $f(m_f)$	without $f(m_f)$	Other results
$[\langle G_{\mu\nu}G^{\mu\nu}\rangle_s/\langle G_{\mu\nu}G^{\mu\nu}\rangle_u]^{1/4}$	1.00	1.20	–
$[\langle \bar{s}s\rangle/\langle \bar{u}u\rangle]^{1/3}$	0.75	0.93	$0.79 \sim 1.03$ [9, 23, 32]
$[\langle \bar{s}\sigma_{\mu\nu}G^{\mu\nu}s\rangle/\langle \bar{u}\sigma_{\mu\nu}G^{\mu\nu}u\rangle]^{1/5}$	0.87	1.00	$0.75 \sim 1.05$ [34, 35, 36, 37, 38]
$m_{0,u}^2 = \langle \bar{u}\sigma_{\mu\nu}G^{\mu\nu}u\rangle/\langle \bar{u}u\rangle$	1.60 GeV^2	1.60 GeV^2	0.8 ± 0.2 [8], 1.4 [11], 2.5 [12] [GeV^2]
$m_{0,s}^2 = \langle \bar{s}\sigma_{\mu\nu}G^{\mu\nu}s\rangle/\langle \bar{s}s\rangle$	1.84 GeV^2	1.92 GeV^2	–

TABLE IV: The ratios of the condensates for the different types of the m_f correction factors. $m_u = 5$ MeV and $m_s = 150$ MeV are used.

directly from the pseudoscalar meson light-cone distribution amplitude within the same framework [39]; $\langle k_T^2\rangle_\pi = 0.23\,\mathrm{GeV}^2$.

4. In the present work, we investigated the various QCD vacuum condensates within the framework of the instanton liquid model, emphasizing the effects of flavor SU(3)-symmetry breaking. The modified improved action elaborated by Musakhanov was used for this purpose. In the modified improved action, the current quark mass appeared explicitly in the denominator of the quark propagator as well as in the dynamical quark mass. Thus, we were able to take into account the current quark mass effects to the QCD condensates. In order to consider the m_s dependence of the dynamical quark mass, we employed the m_f-correction factor. It arises from the resummation of the QCD planar loops in the large N_c limit and is parameterized to satisfy the saddle-point equation [24, 29].

We observed that the gluon condensate is almost independent of the current quark mass when the strange current quark correction factor is introduced, whereas it increases monotonically without it. The quark and mixed condensates were also calculated. The results were consistent with those from other model calculations as well as phenomenological values. In particular, The ratios of the condensates between the strange and up quarks were also investigated: $[\langle\bar{s}\sigma_{\mu\nu}G^{\mu\nu}s\rangle/\langle\bar{u}\sigma_{\mu\nu}G^{\mu\nu}u\rangle]^{1/5}$ and $[\langle\bar{s}s\rangle/\langle\bar{u}u\rangle]^{1/3}$. It turned out that the results are again compatible with other theoretical calculations. The dimensional quantity m_0^2 was also studied: $m_{0,u}^2 = 1.60\,\mathrm{GeV}^2$ and $m_{0,s}^2 = 1.84\,\mathrm{GeV}^2$. In general, the quark and mixed condensates decrease as the current quark mass increases. However, the m_0^2 increases as the current quark mass does, which indicates that the mixed condensate is less sensitive to the current quark mass than the quark condensate. In addition, we also presented the relation of the m_0^2 to the expectation value of the transverse momentum for the leading-twist pion light-cone distribution amplitude.

Though we considered the effects of flavor SU(3)-symmetry breaking on various condensates, we did not take into account those from meson-loop corrections which are known to be of importance. The corresponding investigation is under way.

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